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Gamow-Teller strength function in spherical nuclei

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Abstract. The $\sigma t^{\pm}$ strength distribution as a function of the excitation energy is investigated in the framework of the Quasiparticle Random Phase Approximation. The results are compared with the available experimental data for ^{54}Fe and with the results of recent shell model calculations. It is demonstrated that the full single-particle space has to be used in order to describe the GT strength function correctly.

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1 Introduction

The investigation of the Gamow-Teller (GT) strength function, i.e. the distribution of the strength of the $\sigma_\mu t^+$ or $\sigma_\mu t^-$ operators as a function of the excitation energy is of interest not only in nuclear physics, but it has also important applications in astrophysics. Its precise knowledge is requested for nucleosynthesis problems and in the study of supernova evolution [1]. It also plays an important role in predicting neutrino scattering cross sections used to determine the detector efficiency in solar neutrino flux measurements [2]. In nuclear physics the GT strength function is important in connection with the problem of missing GT strength occurring in the (p,n) charge-exchange reactions at intermediate energies [3], and in connection with the question of a possible renormalization of the axial vector coupling constant of weak interactions in nuclear media, g_A [4]. The GT transitions play a leading role in muon capture, in pion charge-exchange reactions and in other low and medium energy processes in atomic nuclei.

The Random Phase Approximation (RPA) as well as multiparticle shell model calculations give a reasonably description of the shape of the GT strength function at low excitation energies and in the giant resonance region. However, agreement with the experimental data is achieved only after application of a quenching factor of roughly 0.8 to the transition amplitude. From the theoretical point of view one is therefore interested in finding a mechanism which is responsible for the transfer of GT strength towards higher excitation energies and which simultaneously allows one to reproduce the strength distribution in the giant resonance region and below it. Usually the shift of the GT strength to higher excitation energies is explained by introducing an interaction between particle-hole 1p-1h (one-phonon states) and 2p-2h (two-phonon states) [5]. Particle-hole excitations are usually considered on the basis of RPA and their interaction with the 2p-2h states in the framework of the "second random phase approximation" (SRPA) [6,7] or in the fragmentation approach [8]. Additional ground state correlations could be included in the SRPA [9,10] leading to the "extended random phase approximation" (ERPA). Sum rules (energy-weighted moments) for the transition strength of one-body operators can be derived in all the mentioned approximation schemes [9,11]. Based on the properties of these sum rules, in particular the conservation of the zeroth and first energy-weighted moments in RPA, SRPA and the fragmentation approach, we conclude that the inclusion of an interaction between 1p-1h and 2p-2h states is not capable of changing the strength distribution in the desired way. Therefore a satisfactory description of the general features of the GT strength distribution in the framework of RPA seems to be essential. We demonstrate that those parts of the residual interaction coupling single-particle orbits with different radial quantum numbers are essential in order to achieve (at least qualitatively) a reasonable description of the strength distribution. A good overall behaviour of the σt^- strength function at low energies is then obtained and the position and strength of the giant GT resonance can be reproduced. Simultaneously a large fraction of the σt^- strength is shifted towards higher excitations, forming new collective states. These collective states are built up from 1p-1h excitations whose single-particle orbitals belong to different major shells. From this it is concluded that it is absolutely necessary to use a full single-particle space in all GT strength calculations. This statement applies in

particular also to large shell model calculations, where the used single - particle space is limited by the large size of the matrices to be diagonalized.

Since now experimental data for both, σt^+ and σt^- are available, it seemed important to prove whether the proposed mechanism is successful in describing both strength functions simultaneously. The calculations have been performed using a phenomenological residual interaction in the framework of the quasi-particle random phase approximation (QRPA), the latter being an extension of the usual RPA to non-closed shell nuclei.

2 Theory

2.1 The nuclear model

The model Hamiltonian used in the present work has been described in detail in [8,13] and only a short description will be given here. It consists of a mean field part (represented by separate single-particle potential wells of Woods-Saxon shape for neutrons and protons respectively), a superconducting monopole pairing between like particles, and a particle–hole residual interaction in separable form.

$$H = \sum_{\tau=\mathrm{n,p}} (H^\tau_{\mathrm{mean}} + H^\tau_{\mathrm{pair}}) + H_{\mathrm{res.}}, \tag{1}$$

$$H^\tau_{\mathrm{mean}} = \sum_{j_\tau m_\tau} E_{j_\tau} a^\dagger_{j_\tau m_\tau} a_{j_\tau m_\tau}, \tag{2}$$

$$H^\tau_{\mathrm{pair}} = \frac{G_\tau}{4} \sum_{j_\tau m_\tau, j'_\tau m'_\tau} (-1)^{j_\tau - m_\tau + j'_\tau - m'_\tau} \times a^\dagger_{j'_\tau m'_\tau} a^\dagger_{j'_\tau, -m'_\tau} a_{j_\tau, -m_\tau} a_{j_\tau m_\tau} \tag{3}$$

$$H_{\mathrm{res.}} = -2\kappa_1^{01} \sum_\mu Q^\dagger_{1,\mu} Q_{1,\mu}, \tag{4}$$

with

$$Q_{1,\mu} = \sum_{j_p, m_p, j_n, m_n} \langle j_p, m_p | U(r) \sigma_\mu t^- | j_n, m_n \rangle \times a^\dagger_{j_p, m_p} a_{j_n, m_n}. \tag{5}$$

Here κ_1^{01} is the effective coupling constant of the residual interaction, $a^\dagger_{j_p,m_p}$ (a_{j_n,m_n}) is the creation (destruction) operator of a proton (neutron) in the $nljm$-single particle state, $U(r)$ is the radial form factor taken as [12]

$$U(r) = \frac{dW(r)}{dr}, \tag{6}$$

and $W(r)$ is the central part of the single-particle shell model potential.

The case of a strong attractive particle-particle interaction which has been shown to be especially important for low energy σt^+ transitions [13] should be discussed separately. Here we would like to note only that there is a definite contradiction between the description of the β^+ decay of the proton rich spherical nuclei and the β^- decay of neutron rich nuclei when using the particle-hole and particle-particle interactions simultaneously [14].

The diagonalization of the model Hamiltonian is done in two steps. First, we make the transition to the quasiparticle operators by means of the Bogoliubov transformation (for protons and for neutrons separately):

$$\alpha_{j,m} = v_j a_{j,m} + (-1)^{j-m} u_j a^\dagger_{j,-m}. \tag{7}$$

In the next step the charge-exchange 1^+ phonons are introduced

$$\Omega_{\mu,i} = \sum_{j_p, j_n} \{ \psi^i_{j_p,j_n} [\alpha_{j_p} \alpha_{j_n}]_{1,\mu} + (-1)^\mu \phi^i_{j_p,j_n} [\alpha_{j_p} \alpha_{j_n}]^\dagger_{1,-\mu} \} , \tag{8}$$

where

$$[\alpha_{j_p} \alpha_{j_n}]_{\lambda,\mu} \equiv \sum_{m_p, m_n} \langle j_p m_p j_n m_n | \lambda\mu \rangle \alpha_{j_p,m_p} \alpha_{j_n,m_n}$$

and $\langle j_p m_p j_n m_n | \lambda\mu \rangle$ is a Clebsch-Gordan coefficient. The normalization condition for the phonon amplitudes is

$$\sum_{j_p, j_n} \{ \psi^i_{j_p,j_n} \psi^{i'}_{j_p,j_n} - \phi^i_{j_p,j_n} \phi^{i'}_{j_p,j_n} \} = \delta_{i,i'} . \tag{9}$$

The QRPA Hamiltonian is then obtained as

$$\begin{aligned} H \;=\; & \sum_{j_p m_p} \epsilon_{j_p} \alpha^\dagger_{j_p m_p} \alpha_{j_p m_p} + \sum_{j_n m_n} \epsilon_{j_n} \alpha^\dagger_{j_n m_n} \alpha_{j_n m_n} \\ & - \frac{1}{3} \kappa_1^{01} \sum_{i,\mu} \left[(D^i_+)^2 + (D^i_-)^2 \right] \Omega^\dagger_{\mu,i} \Omega_{\mu,i} \end{aligned} \tag{10}$$

where

$$\begin{aligned} & D^i_+ = \sum_q h_q u^+_q g^i_q, \qquad D^i_- = \sum_q h_q u^-_q w^i_q, \\ & g^i_q = \psi^i_{j_p,j_n} + \phi^i_{j_p,j_n}, \qquad w^i_q = \psi^i_{j_p,j_n} - \phi^i_{j_p,j_n}, \\ & u^\pm_q = u_{j_p} v_{j_n} \pm v_{j_p} u_{j_n}, \quad h_q \equiv h(j_p, j_n) \equiv \langle j_p \| U(r) \sigma t^- \| j_n \rangle, \end{aligned}$$

and the ϵ_j are the single-quasiparticle energies. The RPA equations are obtained as usual by first calculating the average value of H over the one-phonon state $\Omega^\dagger_{\mu,i} \Psi_0$ (Ψ_0 is the ground state of a double even nucleus) and then using the variational principle

$$\delta \{ \langle \Psi^*_0 \Omega_{\mu,i} H \Omega^\dagger_{\mu,i} \Psi_0 \rangle - \langle \Psi^*_0 H \Psi_0 \rangle - \omega_i [\sum_q g^i_q w^i_q - 1] \} = 0 \tag{11}$$

The normalization of the phonon amplitudes is used as a subsidiary condition. As a result we get a system of equations

$$\begin{aligned} \epsilon_q g^i_q - \frac{2}{3} \kappa_1^{01} h_q u^+_q D^i_+ - \omega_i w^i_q &= 0 \\ \epsilon_q w^i_q - \frac{2}{3} \kappa_1^{01} h_q u^-_q D^i_- - \omega_i g^i_q &= 0 \end{aligned} \tag{12}$$

defining the phonon amplitudes ψ_q^i, ϕ_q^i ($\epsilon_q \equiv \epsilon_{j_p} + \epsilon_{j_n}$). The amplitudes of the transitions from the ground state to the 1^+ one–phonon excited states are given by

$$b_\mu^+(1^+,i) = \frac{1}{\sqrt{3}} \sum_{j_p,j_n} \langle j_p \| \sigma t^- \| j_n \rangle \times (v_{j_p} u_{j_n} \psi^i_{j_p,j_n} + u_{j_p} v_{j_n} \phi^i_{j_p,j_n}), \quad (13)$$

$$b_\mu^-(1^+,i) = \frac{1}{\sqrt{3}} \sum_{j_p,j_n} \langle j_p \| \sigma t^- \| j_n \rangle \times (u_{j_p} v_{j_n} \psi^i_{j_p,j_n} + v_{j_p} u_{j_n} \phi^i_{j_p,j_n}). \quad (14)$$

The parameters of the single-particle potentials and the monopole pairing constants are taken from [15]. Only one parameter will be varied during the calculations – the effective coupling constant of the residual interaction κ_1^{01}. The calculated GT strength function is given by the running sum

$$S^\pm(E) = \sum_{i:E_i \le E} \sum_\mu |b_\mu^\pm(1^+,i)|^2 \ . \quad (15)$$

2.2 Sum rules

The energy-weighted moments of the strength of the GT transition operators $\sigma_\mu t^\pm$ are defined by

$$S_k^\pm = \sum_{\alpha,\mu} E_\alpha^k \left| \langle \alpha | \sigma_\mu t^\pm | \text{g.s.} \rangle \right|^2 \quad (16)$$

for any integer k. E_α is the excitation energy. It is well known [9] that the difference of the zero order moments $S_0^- - S_0^+$ and the sum of energy-weighted moments $S_1^- + S_1^+$ are conserved when going from RPA to SRPA or the fragmentation approach. One has therefore

$$\left(S_0^- - S_0^+\right)_{\text{RPA}} = \left(S_0^- - S_0^+\right)_{\text{SRPA}} \quad (17)$$

and

$$\left(S_1^- + S_1^+\right)_{\text{RPA}} = \left(S_1^- + S_1^+\right)_{\text{SRPA}}. \quad (18)$$

In the fragmentation approach even simpler relations arise:

$$S_0^\pm\big|_{\text{RPA}} = S_0^\pm\big|_{\text{fragmentation}} \quad (19)$$

and

$$S_1^\pm\big|_{\text{RPA}} = S_1^\pm\big|_{\text{fragmentation}} \quad (20)$$

For details we refer the interested reader to the original literature [11]. We refer here to this approach because it facilitates the following discussion on some discrepancies which we believe to be present in the published literature. The results of this work, however, have been obtained entirely in the framework of the QRPA.

The zeroth energy-weighted moment (17) is the well known Ikeda sum rule [16]. Its value is equal to $3(N-Z)$. The degree to which this sum rule is fulfilled may be used as a test for the completeness of the used single-particle space. In the fragmentation approach it can be shown [11] that S_0 and S_1 are determined by the 1p-1h states only and do not depend on the interaction between 1p-1h states and 2p-2h states or more complex states or on any interaction in the 2p-2h space. This is a direct consequence of separating the ground state wave function from the space of the excited states [11]. The limitation arising from the conservation of the zeroth and first energy-weighted moments can be removed if one considers the ground state to be lying in the same space as the excited states.

It is known that the interaction between 1p-1h and 2p-2h states, responsible for the width of the giant resonance, causes some redistribution of the transition strength over the excitation energies [17,18]. The conservation of zeroth and first moments has, however, a severe consequence. We discuss this in the framework of the fragmentation approach, where $S_0^\pm$ and $S_1^\pm$ are separately conserved, eqs. (19) and (20). Due to the fact that the total transition strength $S_0^\pm$ and the energy centroid $S_1^\pm/S_0^\pm$ are simultaneously conserved, we have the following situation. If on behalf of an interaction between 1p-1h states and more complex states a large fraction of the strength of the giant GT resonance could be shifted to higher energies, then some strength has necessarily to be shifted into the low energy region in order to conserve the first moment, $S_1^\pm$. As a result the strength distribution in the giant resonance region and below it would change completely. Such an effect has been found by shell model calculations of the GT strength function [19]. Enlarging the excited states space by including 2p-2h configurations without changing the ground state, the authors of [19] observed exactly the above described effect. The authors of [7] performed GT strength function calculations for the nuclei ^{48}Ca, ^{90}Zr and ^{208}Pb in the framework of RPA and SRPA, using a realistic two-body interaction based on a Brueckner G-matrix for nuclear matter. In order to apply the nuclear matter G-matrix to finite nuclei, the local density approximation was used. In case of ^{208}Pb the authors of [7] found already at the level of RPA a large fraction of strength above 20 MeV excitation energy originating from a coupling of the GT resonance with high lying $2\hbar\omega$ particle-hole 1^+ excitations. The main conclusion of [7] is that a large fraction of the total GT strength is shifted towards higher excitation energies mainly due to the interaction between 1p-1h and 2p-2h states. Simultaneously, the strength in the giant resonance region and below it is reduced. The energy of the GT resonance is practically not decreased and for ^{208}Pb shifted even to higher energies. Due to the large neutron excess in heavy nuclei the σt^- strength measured in (p,n) reactions is much stronger than the σt^+ strength related to (n,p) reactions. Therefore S_0^+ is only a small fraction of S_0^- and one can assume without making too big an error that S_0^- and S_1^- practically do not change when going from RPA to SRPA. One has then $S_{0,1}^-|_{\text{RPA}} \approx S_{0,1}^-|_{\text{SRPA}}$. Then, however, the arguments presented above for the fragmentation approximation apply and one sees immediately that there is a contradiction between the results quoted in [7] and the conservation of S_0 and S_1.

The arguments presented above have been the basic motivation to undertake the present work, aiming to un-

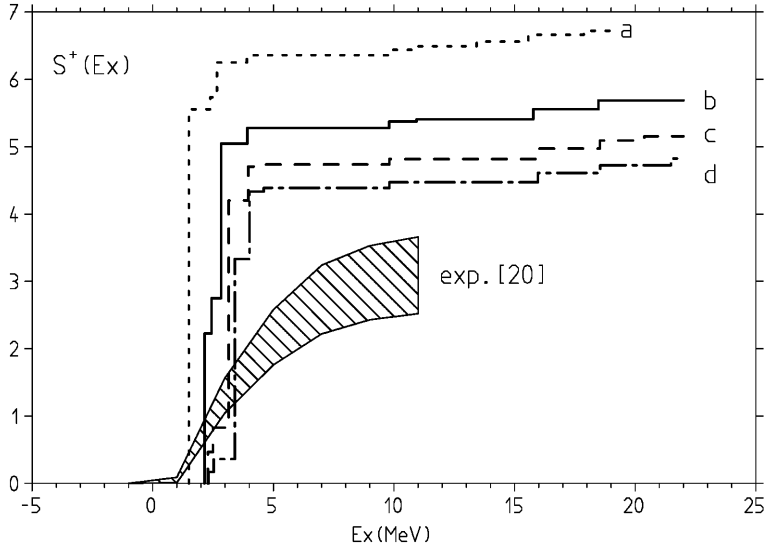

Fig. 1. Running sum $S^+(E)$ for the σt^+ transition operator as a function of the excitation energy of the residual nuclei. The shaded area represents the experimental strength function for the ^{54}Fe(n, p)^{54}Mn reaction [20]. a), b), c) and d) represent QRPA calculation for various values of κ_1^{01} (a: $-0.23/A$, b: $-0.43/A$, c: $-0.63/A$ and d: $-0.83/A$)

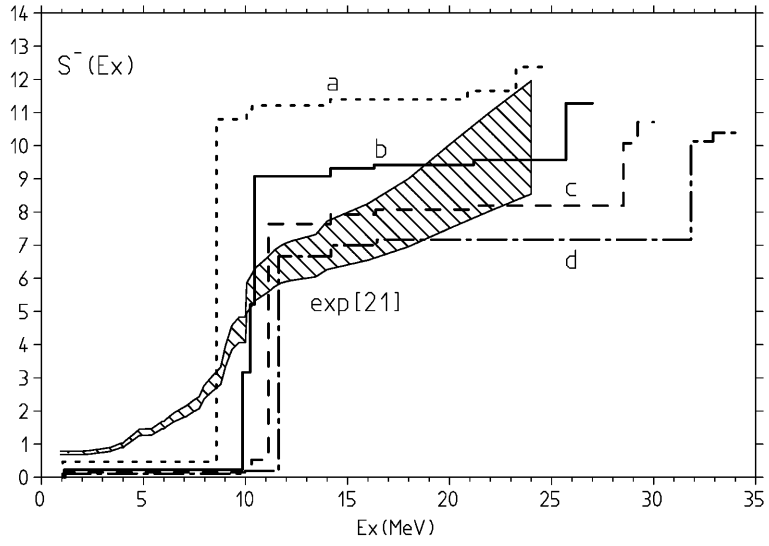

Fig. 2. Running sum $S^-(E)$ for the σt^- transition operator as a function of the excitation energy of the residual nuclei. The notation is the same as in Fig. 1. The experimental data for the ^{54}Fe(p, n)^{54}Co reaction are from [21]

derstand the GT strength function problematic at the RPA level.

3 Results and discussions

Next we present the results of our calculations of the GT strength function (15) for ^{54}Fe and compare them with detailed experimental studies of the σt^+ and σt^- strength functions by means of the (p,n) [20,21] and (n,p) [20] reactions on ^{54}Fe. These experimental results together with our theoretical evaluation are shown in Figs. 1, 2 and 3.

Figure 1 shows the experimental running sum $S^+(E)$ from the ^{54}Fe(n, p)^{54}Mn reaction [20] in comparison with the theoretical results calculated for different values of κ_1^{01} ($-0.23/A$, $-0.43/A$, $-0.63/A$ and $-0.83/A$). The distribution of the σt^+ strength at low excitation energies is determined by the positions of the two–quasiparticle states. There is one collective state which absorbs the main part of the transition strength. With increasing absolute value of the effective interaction constant $|\kappa_1^{01}|$ the collective state is shifted towards higher excitation energies and its strength decreases. Simultaneously the total transition strength becomes smaller. The calculated and measured strength distributions [20] are in qualitative agreement. It should be mentioned that in the low excitation energy region a much richer experimental spectrum is observed than calculations show. The total σt^+ strength measured in the (n,p) reaction up to excitation energies of 10 MeV is equal to 3.1 ± 0.6 (all energies are measured with respect to the ground state of the residual nuclei). The calculated QRPA strength for this energy range lies between 4.2 and 6.5 depending on the value of $|\kappa_1^{01}|$. A recent shell model calculation [22] needs a quenching factor of 0.77 for the σt operators in order to reproduce the experimentally observed transition strength. The work of [22] contains also a review of previous calculations of $S^{\pm}$ in ^{54,56}Fe. Various shell model calculations and the QRPA with different residual interactions give usually higher values for S^+ than those obtained in the present work. An exception is the

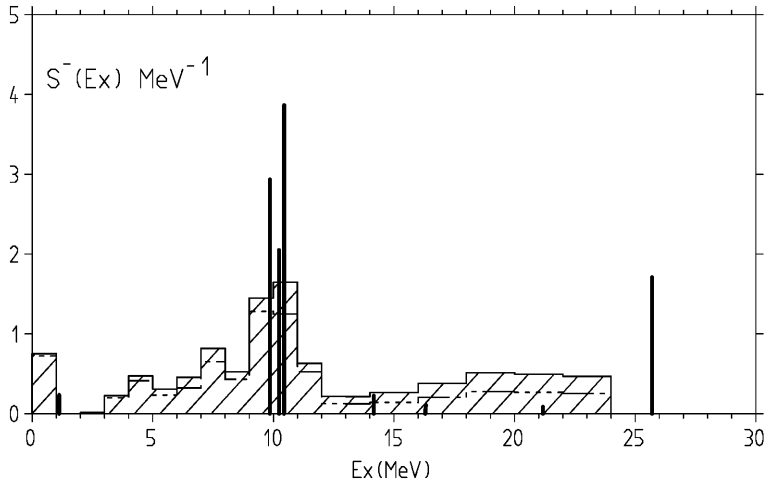

Fig. 3. $\sigma_\mu t^-$ strength function as a function of excitation energy. Experimental data (shaded area) for the ^{54}Fe(p, n)^{54}Co reaction are from [21]. The theoretical calculation for $\kappa_1^{01} = -0.43/A$ corresponds to curve b) of Fig. 2

QRPA case with the particle-particle interaction of [13] which gives $S^+ = 4.2$. The relative low value of S^+ in our present calculation is due to the residual interaction used. The radial form factor $U(r)$ varies rapidly with radius and differs from zero only in the surface region. Therefore the matrix elements between single–particle states with different radial quantum numbers contribute appreciably. In this way a mixing not only between the usual spin–orbit partners of the valence shell occurs (as should be the case in shell model calculations), but is present also between all the other single particle states. The influence of this mixing becomes especially important in the case of σt^- transitions, when the Gamow-Teller resonance can be excited.

The results of our calculations of $S^-(E)$ for the same set of values of κ_1^{01} are shown in Fig. 2 together with the GT strength function measured in the ^{54}Fe(p, n)^{54}Co reaction [20,21] (data are taken from Tables 1 and 3 of [21]). The results of [20] are close to those of [21] and have larger error bars. Figure 3 shows a comparison between the experimental and theoretical ($\kappa_1^{01} = -0.43/A$) strength functions plotted as a function of excitation energy.

The following observations can be made:
(i) a collective GT state located above the conventional GT giant resonance appears if the residual interaction is strong enough. This state is formed on the basis of two–quasiparticle states in which neutron and proton quasiparticles occupy levels with different radial quantum numbers;
(ii) as $|\kappa_1^{01}|$ increases, this collective state absorbs a steadily-increasing part of the total σt^- strength and is shifted gradually to higher excitation energies;
(iii) accordingly, the conventional giant resonance around 10 MeV loses part of its transition strength and is shifted only slightly towards higher excitation energies. The reason for this positional stability are additional two-quasiparticle poles appearing in the QRPA secular equation located above those forming the giant GT resonance. It should be mentioned that the standard way of obtaining the value of the effective coupling constant from the position of the giant resonance meets difficulties in this case.

Introducing the residual interaction in its present form allows to describe the main features of the $\sigma t^\pm$ strength distribution and to reproduce the experimentally observed GT transition strength without any additional effective charges [23] or quenching factors [22].

For all values of κ_1^{01} the difference between the total σt^- and σt^+ strength is

$$S_0^- - S_0^+ = 5.61.$$

This value is somewhat less than $S_0^- - S_0^+ = 6.0$ predicted by the Ikeda sum rule [16] for ^{54}Fe. The origin of this difference is mainly the small nonorthogonality of the neutron and proton single–particle wave functions due to the difference in the single–particle potential wells.

4 Conclusion

The present work leads to the following conclusions. It is possible to describe in the QRPA simultaneously the strength functions of $\sigma t^\pm$ transitions in ^{54}Fe. The use of a separable nonlocal residual interaction allows a qualitative description of the σt^+ strength function. The calculated S^+ strength below 10 MeV is between 4.3 and 5.4 and to be compared with 3.1 ± 0.6 obtained experimentally in the (n,p) reaction [20]. The calculated σt^- strength function agrees with the strength function obtained from the (p,n) reaction in the low excitation energy region and in the giant resonance region without any quenching factors. The rest of the GT strength is absorbed by high-lying collective 1^+ states formed around two-quasiparticle states having different number of nodes in the radial part of their single-particle wave functions, i.e. belonging to different major shells. This is basically the reason why it is important to use a large single-particle space in the calculation of the σt^- strength function.

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